VISCOUS FRICTION AND HEAT FLUX FOR A PARTIALLY IONIZED MEDIUM FLOWING IN A PLANE CHANNEL WITH ALLOWANCE FOR ANISOTROPY OF THE TRANSPORT COEFFICIENTS

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In [1, 2], on the assumption that the magnetic Reynolds number was small, the author obtained exact expressions for the stationary complex velocity $v(z) = u_x - iu_y$ and temperature $T(z)$ of an ionized medium moving between parallel plates under the influence of a constant pressure drop P_x in a strong uniform transverse magnetic field $B_z \equiv B_0$. The height of the channel is $2a$, the walls are kept at the constant temperature $T(\pm a) = 0$, and there is no external electric field.

The medium is assumed to be incompressible (ρ = const) and its degree of ionization is constant (s = const).

Then for viscous friction stress we have

$$\tau^2 = \pi_{xz}^2 + \pi_{yz}^2. \tag{1}$$

Here the components of the viscous stress tensor π_{xz} and π_{yz} have the form [3]

$$\pi_{xz} = -\eta^{(2)}\frac{\partial u_x}{\partial z} - \eta^{(4)}\frac{\partial u_y}{\partial z}, \qquad \pi_{yz} = -\eta^{(2)}\frac{\partial u_y}{\partial z} + \eta^{(4)}\frac{\partial u_x}{\partial z}. \tag{2}$$

Expressions for the viscosity coefficients $\eta^{(2)}$ and $\eta^{(4)}$ are presented in [1].

Substituting (2) in (1) and going over to the nondimensional form, taking as the scales of τ, u and z the quantities and a, respectively, for the square of the local viscous friction coefficient we get

$$c_f^2 = \frac{4\tau^2}{\rho^2 U_0^4} = \frac{4}{R^{(\circ)2}}\left[\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial u_y}{\partial z}\right)^2\right]\frac{1 + {}^4/_9(\omega_i\tau_i\theta)^2(\eta_a/\eta^{(\circ)})^2}{1 + {}^4/_9(\omega_i\tau_i\theta)^2}\left(R^{(\circ)} = \frac{U_0 a\rho}{\eta^{(\circ)}}\right). \tag{3}$$

Here $R^{(\circ)}$ is the Reynolds number, and $\eta^{(\circ)}$ and η_a are the viscosity coefficients of the partially ionized medium as a whole and of the "isolated" neutrals in the case when the magnetic field is equal to zero [3]. The dimensionless parameter $\omega_i\tau_i\theta$ characterizes the anisotropy of the viscosity coefficients (ω_i is the ion cyclotron frequency, $\tau_i\theta$ is related to the time of all possible ion collisions).

If in the expression for the stationary velocity we separate the real and imaginary parts and substitute in (3), we finally get for the surface friction at the upper wall (z = 1)

$$\frac{1}{P_x^2}c_{f+}^2 = \frac{4(\Delta_1^{*2}+\Delta_2^{*2})(r_1^2+r_2^2)(\mathrm{th}^2 r_1 + \mathrm{tg}^2 r_2)[1 + {}^4/_9(\omega_i\tau_i\theta)^2(\eta_a/\eta^{(\circ)})^2]}{M^{(\circ)4}(1 + \mathrm{th}^2 r_1\,\mathrm{tg}^2 r_2)[1 + {}^4/_9(\omega_i\tau_i\theta)^2]}, \tag{4}$$

$$\Delta_1^* = 1 + \frac{2(1-s)^2}{1+Zs}\omega_i\tau_{ia}\omega_e\tau_0, \qquad \Delta_2^* = \frac{\omega_e\tau_0}{1+Zs}, \quad r_{1,2} = \left(\frac{\sqrt{m_1^2+m_2^2} \pm m_1}{2(m_1^2+m_2^2)}\right)^{1/2}, \tag{5}$$

$$m_1 = \frac{1}{M^{(\circ)2}[1 + {}^4/_9(\omega_i\tau_i\theta)^2]}\left[1 + \frac{4}{9}(\omega_i\tau_i\theta^2)\frac{\eta_a}{\eta^{(\circ)}} - \frac{2}{3}\omega_e\tau_0\omega_i\tau_i\theta\left(1 - \frac{\eta_a}{\eta^{(\circ)}}\right) + \right.$$
$$\left. + 2(1-s)\omega_e\tau_0\omega_i\tau_{ia}\left(1 + \frac{4}{9}(\omega_i\tau_i\theta)^2\frac{\eta_a}{\eta^{(\circ)}} - \frac{\eta_i^{(\circ)}}{\eta^{(\circ)}}\right)\right],$$

$$m_2 = \frac{1}{M^{(\circ)2}[1 + {}^4/_9(\omega_i\tau_i\theta)^2]}\left\{\omega_e\tau_0\left[1 + \frac{4}{9}(\omega_i\tau_i\theta)^2\frac{\eta_a}{\eta^{(\circ)}}\right] + \right.$$
$$\left. + \frac{2}{3}\omega_i\tau_i\theta\left[1 - \frac{\eta_a}{\eta^{(\circ)}} + 2(1-s)\omega_i\tau_{ia}\omega_e\tau_0\left(1 - \frac{\eta_a}{\eta^{(\circ)}} - \frac{\eta_i^{(\circ)}}{\eta^{(\circ)}}\right)\right]\right\}, \tag{6}$$

$$M^{(\circ)} = B_0 a\left(\frac{\sigma_0}{\eta^{(\circ)}}\right)^{1/2} \quad \frac{1}{\tau_0} = \frac{1}{\tau_{ei}} + \frac{1}{\tau_{ea}}.$$

Here $M^{(\circ)}$ is the Hartmann number, σ_0 is the conductivity of the medium, $\eta_i^{(\circ)}$ is the ion viscosity coefficient $B_0 = 0$, Z is the charge number, ω_e is the electron cyclotron frequency, $\tau_{\alpha\beta}^{-1}$ is the effective collision frequency for particles of types α and β.

Note that the relations $\eta_a/\eta^{(\circ)}$ and $\eta_i^{(\circ)}/\eta^{(\circ)}$ entering into (4) depend significantly on the degree of ionization s. To establish this dependence requires a special study; however, for the extreme cases of weakly ionized and completely ionized medium it is easy to obtain [3]

$$(\eta_a/\eta^{(0)})_{s=1} = (\eta_i^{(0)}/\eta^{(0)})_{s\ll 1} = 0, \qquad (\eta_a/\eta^{(0)})_{s\ll 1} = (\eta_i^{(0)}/\eta^{(0)})_{s=1} = 1. \tag{7}$$

By virtue of the geometry of the problem (heat transfer is realized along the magnetic field), the heat flux to the plate has the same form as in isotropic magnetohydrodynamics

$$q_z^w = -\lambda^T \left(\frac{\partial T}{\partial z}\right)_{z=a} . \tag{8}$$

Here λ^T is the heat conductivity of the medium in the absence of a magnetic field [3]. The effect of the magnetic field on the heat flux is therefore realized only through the temperature gradient. An expression for the latter is easily obtained from the formula for the stationary temperature found in [2]. Carrying out the necessary transformations, in dimensionless form we get

$$\left(\frac{dT}{dz}\right)_{z=1} = \left(\frac{dT_\eta}{dz}\right)_{z=1} + \left(\frac{dT_\sigma}{dz}\right)_{z=1} \tag{9}$$

$$\frac{-1}{\gamma P_x^2 R^{(0)^2} N_{\mathrm{Pr}}^{(0)}} \left(\frac{dT_\eta}{dz}\right)_{z=1} = \frac{(\Delta_1^{*2} + \Delta_2^{*2})(r_1^2 + r_2^2)[1 + {}^4/_9 (\omega_i\tau_i\theta)^2 \eta_a/\eta^{(0)}}{4M^{(0)^4}(\mathrm{ch}^2 r_1 \cos^2 r_2 + \mathrm{sh}^2 r_1 \sin^2 r_2)[1 + {}^4/_9 (\omega_i\tau_i\theta)^2} \left(\frac{\mathrm{sh}\, 2r_1}{r_1} - \frac{\sin 2r_2}{r_2}\right)$$

$$\left(N_{\mathrm{Pr}} = \frac{\eta^{(0)} c_{pa}}{\lambda^T}, \ \gamma = \frac{U_0^2}{c_{pa} T^*} \right) \tag{10}$$

$$\frac{-1}{\gamma P_x^2 R^{(0)^2} N_{\mathrm{Pr}}^{(0)}} \left(\frac{dT_\sigma}{dz}\right)_{z=1} = \frac{\Delta_1^*}{M^{(0)^2}} +$$

$$+ \frac{1}{2M^{(0)^3}(\mathrm{ch}^2 r_1 \cos^2 r_2 + \mathrm{sh}^2 r_1 \sin^2 r_2)} \{\mathrm{sh}\, 2r_1 [r_1(m_2\Delta_2^* - m_1\Delta_1^*) - r_2(m_2\Delta_1^* + m_1\Delta_2^*)] - \tag{11}$$

$$- \sin 2r_2 [r_1(m_2\Delta_1^* + m_1\Delta_2^*) + r_2(m_2\Delta_2^* - m_1\Delta_1^*)]\} + \frac{1}{\gamma P_x^2 R^{(0)^2} N_{\mathrm{Pr}}^{(0)}} \left(\frac{dT_\eta}{dz}\right)_{z=1}$$

The symbols η and σ denote the parts of the gradient characterizing viscous dissipation and Joule heating at the upper wall, $N_{pr}^{(\circ)}$ is the Prandtl number, C_{pa} is the heat capacity of unit mass of neutrals, T^* is the characteristic temperature.

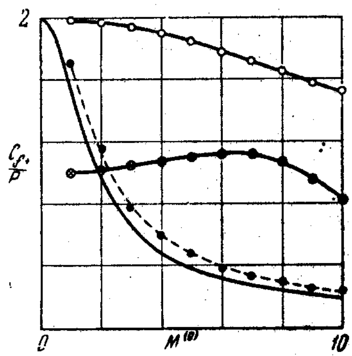

Fig. 1

Below we present the results of a numberical study of the dependence of the expressions obtained for the viscous friction coefficient (4) and the dimensionless temperature gradients (9)-(11) on the Hartmann number $M^{(\circ)}$ for different values of the anisotropy parameters of the electrons and ions. The results are plotted in Figs. 1-4, where the following notation has been introduced:

$$\theta = \vartheta \left[\frac{dT}{dz}\right]_{z=1}, \quad \theta_\sigma = \vartheta \left[\frac{dT_\sigma}{dz}\right]_{z=1},$$

$$\theta = \vartheta \left[\frac{dT_\eta}{dz}\right]_{z=1}, \quad \vartheta = \frac{-1}{\gamma P_x^2 R^{(\circ)^2} N_{\mathrm{Pr}}^{(\circ)}} .$$

The curves 1-5 in the figures correspond to the following values of the parameters:

$$
\begin{array}{lll}
1\ \omega_e\tau_0 \ll 1, & \omega_i\tau_i\theta \ll 1, & s \in [0, 1], \\
4\ \omega_e\tau_0 = 40, & \omega_i\tau_{ia} = 1, & s \ll 1, \\
5\ \omega_e\tau_0 = 40, & \omega_i\tau_i\theta = 1, & s = 1, \\
2\ \omega_e\tau_0 = 1, & \omega_i\tau_i \ll 1, & s \ll 1, \\
3\ \omega_c\tau_0 = 1, & \omega_i\tau_i \ll 1, & s = 1.
\end{array}
$$

In the calculations it was assumed that Z = 1. Analysis of the data obtained enables one to draw the following conclusions.

1. When $\omega_e\tau_0 \ll 1$ (isotropic magnetohydrodynamics) an increase in $M^{(0)}$ leads to a fall in the viscous friction coefficient, which has a maximum at $M^{(0)} = 0$, i. e., in the purely hydrodynamic case (Fig. 1). This fall is a consequence of the known decrease in the velocity gradient at the wall with increase in Hartmann number owing to the retarding effect of the ponderomotive force. The decrease in viscous friction at the wall leads also to a decrease in viscous dissipation with increase in $M^{(0)}$ (Fig. 3). The Joule heat flux to the wall has a somewhat different character (Fig. 4). Increasing from 0 at $M^{(0)} = 0$, it reaches a maximum at $M^{(0)} \approx 1.6$ and only then begins to fall. However, the total heat flux to the upper wall (Fig. 2) decreases monotonically with increase in $M^{(0)}$, while for $M^{(0)} < 1.6$ the contribution of viscous heat exceeds the contribution of Joule heat, and at $M^{(0)} > 1.6$ the reverse picture is observed.

Note that a study of the heat flux to the wall in the Hartmann flow regime, when the electric field is not zero, was made in [4, 5].

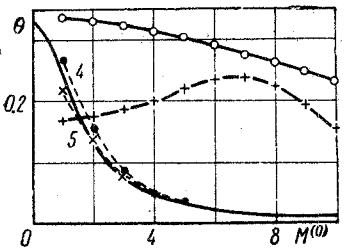

Fig. 2

Note that in [5] an incorrect conclusion was drawn concerning the increase in heat flux with increase in Hartmann number when the latter is large. The authors did not take into account the dependence of the fluid velocity in the middle plane of the channel (z = 0) on the Hartmann number. Taking this factor into account also leads to a fall in heat flux with increase in Hartmann number, and in this case when $M^{(0)} \gg 1$ the flux decreases as $M^{(0)-1}$. In the regime with no electric field the decrease is more intense, namely, as $M^{(0)-2}$.

2. As may be seen from the graphs, taking into account the conductivity anisotropy when $\omega_e\tau_0 = 1$, $\omega_i\tau_i\theta \ll 1$ does not have much effect on the behavior of the viscous friction and the heat flux at the wall. Note, however, that for weak ionization of the medium ($s \gg 1$) both the quantity C_{f+} and the derivative θ for the same values of $M^{(0)}$ are greater than in the case of isotropic magnetohydrodynamics. An increase in the degree of ionization leads to a decrease in C_{f+}, $[dT/dz]_{z=1}$, $[dT_\eta/dz]_{z=1}$ and an increase in $[dT_\sigma/dz]_{z=1}$. Thus

$$C_{f+}|_{s=1/2} = 0.8512\, C_{f+}|_{s \ll 1},\quad C_{f+}|_{s=1} = 0.7918\, c_{f+}|_{s\ll 1},$$

$$\left(\frac{dT_\eta}{dz}\right)_{z=1}^{s=1/2} = 0.7222\left(\frac{dT_\eta}{dz}\right)_{z=1}^{s\ll 1},\qquad \left(\frac{dT_\eta}{dz}\right)_{z=1}^{s=1} = 0.625\left(\frac{dT_\eta}{dz}\right)_{z=1}^{s\ll 1}.$$

The latter is linked with intensification of the current density, an increase in the retarding action of the ponderomotive force, and, hence, with a decrease in the velocity gradients at the duct wall.

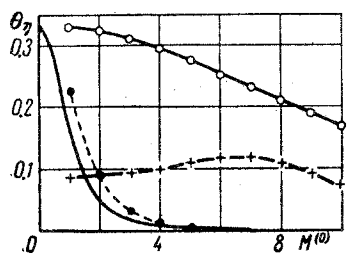

Fig. 3

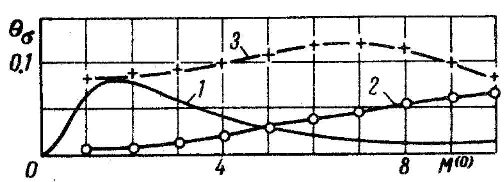

Fig. 4

3. Taking into account the effects of Larmor precession of both electrons and ions sharply changes the form of the curves characterizing viscous friction and the heat flux at the wall. In this case the degree of ionization also has an important influence.

When s ≫ 1 for identical Hartmann numbers at $\omega_i \tau_{ia} = 1$, $\omega_e \tau_0 = 40$ one observes a sharp increase both in surface friction and in the temperature gradient at the upper wall, as compared with the previous cases. This is connected, in particular, with a weakening of the current density owing to inclusion of the effect of ion slip relative to the neutrals. Although a decrease in c_{f+} and $[dT / dz]_{z=1}$ with increase in $M^{(\circ)}$ is still observed, the intensity of this decrease is reduced. Up to very large $M^{(\circ)}$ the contribution to the heat flux from viscous dissipation considerably exceeds the contribution from Joule heat (weak current).

In the other extreme case, when $s = 1$, c_{f+} and $[dT / dz]_{z=1}$ increasing with increase in $M^{(\circ)}$, pass through a maximum and only then begin to decrease monotonically with further increase in $M^{(\circ)}$. Here the contributions to the heat flux made by viscous dissipation and Joule heating are almost equivalent.

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